647510 **MSC3MatC302x** **Seat No :________**

M.Sc. (Maths) Semester - 3 (*CBCS*) Examination

DEC/JAN. - 2020

Functional Analysis (Core)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figure to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1(a) Answer each of the followings. **[4]**

(1) Justify that the set $\{(1,0),(1,2),(-1,3)\}$ is linearly dependent in $\mathbb{R}^2$.

(2) If a linear map $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined as $T(x,y) = (x+y, y-x, -x-y)$ then find the null space of T.

Q.1(b) Answer any two questions. **[10]**

(1) Prove that every finite dimensional normed space is complete.

(2) Show that the set of complex numbers $\mathbb{C}$ is a Banach space with appropriate norm on it.

(3) Show that the dual space of l^1 is l^∞.

Q.2(a) Answer each of the followings. **[4]**

(1) Define absolute convergent series in a normed space and give one example of it.

(2) Explain l^p $(1 \le p < \infty)$ spaces with a norm on it.

Q.2(b) Answer any two questions. **[10]**

(1) Let (x_n) be a weakly convergent sequence in a normed space X. Prove that the weak limit x of (x_n) is unique and every subsequence of (x_n) converges weakly to x.

(2) Show that any two norms on a finite dimensional normed space are equivalent.

(3) Show that every convergent sequence is Cauchy in a normed space. Also, give counter example for the converse.

Q.3(a) Answer each of the followings. **[4]**

(1) Give any two equivalent norms on $\mathbb{R}^2$.

(2) Define $BV[a,b]$ with a norm on it.

Q.3(b) Answer any two questions. **[10]**

(1) State and prove closed graph theorem.

(2) State the Hahn-Banach theorem for normed spaces with illustration.

(3) Let X and Y be normed spaces and $T: X \to Y$ be a bounded linear map. Then show that the adjoint operator $T^\times: Y' \to X'$ is linear, bounded and $||T^\times|| = ||T||$.

Q.4(a) Define reflexive spaces and show that l^1 is not reflexive. **[4]**

Q.4(b) Answer any two questions. **[10]**

(1) Prove that every Hilbert space is reflexive.

(2) State and prove Bessel inequality for an inner product space X.

(3) Sate and prove Riesz representation theorem for bounded linear functional on a Hilbert space H.

Q.5(a) Define normal and self-adjoint operator. Also, give an example of a normal operator which is not self-adjoint. **[4]**

Q.5(b) Answer any two questions. **[10]**

(1) Let $T: H \to H$ be a bounded linear operator on a Hilbert space H. Then prove the followings:

(i) If T is self-adjoint, then $\langle Tx, x\rangle$ is real for all $x \in H$.

(ii) If H is complex and $\langle Tx, x\rangle$ is real for all $x \in H$, then the operator T is self-adjoint.

(2) A bounded linear operator $P: H \to H$ on a Hilbert space H is a projection if and only if P is self-adjoint and idempotent.

(3) Let P_1 and P_2 be projections on a Hilbert space H. Then prove that the sum $P = P_1 + P_2$ is a projection on H if and only if $Y_1 = P_1(H)$ and $Y_2 = P_2(H)$ are orthogonal.

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